The Composite Fermion: A Quantum Particle and Its Quantum Fluids

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Discovery of new particles is not usually associated with condensed matter physics, because, at one level, we already know all the particles that go into the Hamiltonian — namely, electrons and ions. However, it is a most profound and wonderful fact of nature — indeed the very reason why physics can make progress at many different levels — that strongly interacting particles reorganize themselves to become more weakly coupled particles of a new kind. Often these are simple bound states of the old particles, but sometimes they are fantastically complicated collective objects (for example, solitons), that nonetheless behave as legitimate particles with well defined charge, spin, statistics, and other properties we attribute to particles.

In fact, these new particles are the true particles of the system in question, because it is conceptually sensible to reserve the title "particle" for nearly independent objects. Once we have identified the true particles of a system, phenomena that would be utterly mysterious, difficult, or impossible to understand in terms of the old particles become simply comprehensible as properties of almost free particles. That is why condensed matter systems are often described in terms of phonons, magnons, Landau quasiparticles, or Cooper pairs, rather than electrons and ions.

This article concerns electrons confined to two dimensions, as realized, for example, at the interface between two semiconductors. In strong strong transverse magnetic fields and at sufficiently low temperatures, this system exhibits absolutely marvelous experimental properties that are entirely unexpected and inexplicable when it is viewed as a collection of weakly interacting electrons.

So, what are the true particles of this two-dimensional electron system? It happens that electrons effectively "swallow" all or a substantial fraction of the external magnetic field, thus transforming themselves into particles called "composite fermions."[1,2] Numerous properties of composite fermions and the quantum fluids they form have been investigated over the last decade. Their Fermi sea, Shubnikov-de Haas oscillations, cyclotron orbits and quantized Landau levels have been observed; their charge, spin, statistics, mass, magnetic moment, and thermopower have been measured; and they have been bounced around like a billiard ball in mesoscopic experiments[3].

The composite fermion has not only helped explain and predict dramatic phenomena, but also motivated a zero parameter microscopic theory that is practically exact. It will be impossible, in this limited space, to do justice to the vast and growing body of work in the field. So this article will concentrate only on some of the most basic facts, pointing the interested reader to review articles or the most recent papers for further information.

Basic concepts

In the presence of a magnetic field B transverse to a two-dimensional system of electrons, the kinetic energy of an electron is quantized into discrete levels, called Landau levels (Fig. 1). The degeneracy of each Landau level per unit area — that is to say, its maximum population per unit area — is $|B|/\Phi_0$, where $\Phi_0 \equiv hc/e$ is the elementary quantum of magnetic flux. This degeneracy implies that the number of occupied Landau levels, called the filling factor, is $\nu = \rho\Phi_0/|B|$, where ρ is the two-dimensional density of electrons.

In a sufficiently strong magnetic field, when $\nu < 1$, all electrons can be accommodated in the lowest Landau level, and to good approximation, one can neglect any mixing between Landau levels. The kinetic energy is then an irrelevant constant and the Hamiltonian is simply given by the

Coulomb potential of the electron assemblage:

$$\hat{\mathcal{H}} = \frac{1}{2}\sum_{j\neq k}\frac{e^2}{|\mathbf{r}_j - \mathbf{r}_k|}. \tag{1}$$

The ultimate goal of theory is to solve the Schrödinger equation $\hat{\mathcal{H}}\Psi = \mathcal{E}\Psi$, as a function of ν in the Hilbert space of the lowest Landau level states. Considering that we are dealing with a macroscopic system of interacting electrons, it should not come as a surprise that we do not know the exact solution. To make matters worse, the standard approximate perturbative strategies are doomed by the absence of any small parameter, the interaction energy being the only energy scale in the problem. Nonetheless, a trail of experimental clues has guided us to wave functions that are accurate and faithful representations of the exact eigenstates.

These wavefunctions reveal the simple physics of the problem, namely the formation of the composite fermion. The composite fermion was originally introduced in 1989 to explain the fractional quantum Hall effect, discovered in 1982 by Daniel Tsui, Horst Stormer, and Arthur Gossard. But subsequent work has shown that it describes a superstructure that encompasses other phenomena as well.

The quickest way to introduce the composite fermion is through the following series of steps, which I will call the "Bohr theory" of composite fermion because it obtains some of the essential results with the help of an oversimplified but useful picture. (If questions occur to you as we go along, please be patient ... some may be answered in the paragraphs that describe the microscopic Schrödinger wave functions.) The outcome is that strongly interacting electrons in a strong magnetic field B transform into weakly interacting composite fermions in a weaker magnetic field B^* given by

$$B^* = B - 2p\rho\Phi_0 \tag{2}$$

where $2p$ is an even integer. Equivalently, one can say that electrons at filling factor ν convert into composite fermions with filling factor $\nu^* \equiv \rho\Phi_0/|B^*|$, given by

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}\,. \tag{3}$$

The minus sign corresponds to situations when B^* points antiparallel to B.

Start by considering interacting electrons in the transverse magnetic field B. Now attach to each electron an infinitely thin, massless magnetic solenoid carrying $2p$ flux quanta pointing antiparallel to B, turning it into a composite fermion. The composite fermion here is modeled as the bound state of an electron and $2p$ flux quanta. Such a conversion preserves the minus sign associated with an exchange of two particles, because the bound state of an electron and an even number of flux quanta itself is a fermion. Hence the name. It also leaves the Aharonov-Bohm phase factors associated with all closed paths unchanged, because the additional phase factor due to a flux $\Phi = 2p\Phi_0$ is $e^{2\pi i\Phi/\Phi_0} = 1$. In other words, the attached additional flux is unobservable, and the new problem formulated in terms of composite fermions is identical to the one with which we began.

What have we gained? Well, a "mean-field approximation" now suggests itself in which the new attached field is smeared to produce an additional uniform magnetic field $-2p\rho\Phi_0$. With that addition, we get the net magnetic field B^* of Eq. (2). The net effect, in a sense, is that each electron has absorbed $2p$ flux quanta from the external field to become a composite fermion, that experiences only the residual magnetic field B^* (Fig 2).

The crucial point is this. The many-particle ground state of electrons at $\nu < 1$ was highly degenerate in the absence of interaction, with *all* lowest Landau level configurations having the same energy. But now, the degeneracy of the composite fermion ground state at the corresponding

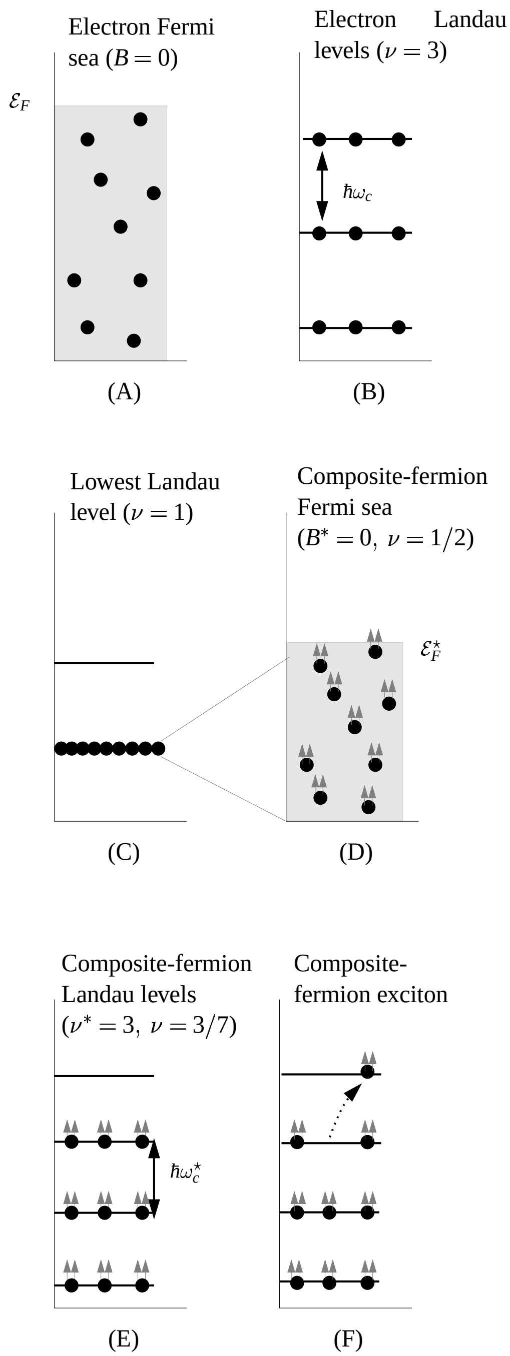

Figure 1. Evolution of a two-dimensional electron system as the transverse magnetic field B is increased. For independent electrons, the Fermi sea (A) (filled to Fermi energy $\mathcal{E}_F$) at $B = 0$ splits into Landau levels (B) separated by the cyclotron energy. The lowest electronic Landau level (C) is split by interactions into energy levels of composite fermions, with attached flux quanta, which fill a composite-fermion Fermi sea at $\nu = 1/2$ (D) and occupy composite-fermion Landau levels (E) at other filling factors. A jump out of such a level (F) creates an exciton, a neutral particle-hole excitation. $\mathcal{E}_F^*$ and $\hbar\omega_c^*$ are the composite-fermion Fermi and cyclotron energies, respectively. At still higher fields, this scenario (D-F) repeats itself, but now with composite fermions carrying four or more flux quanta.

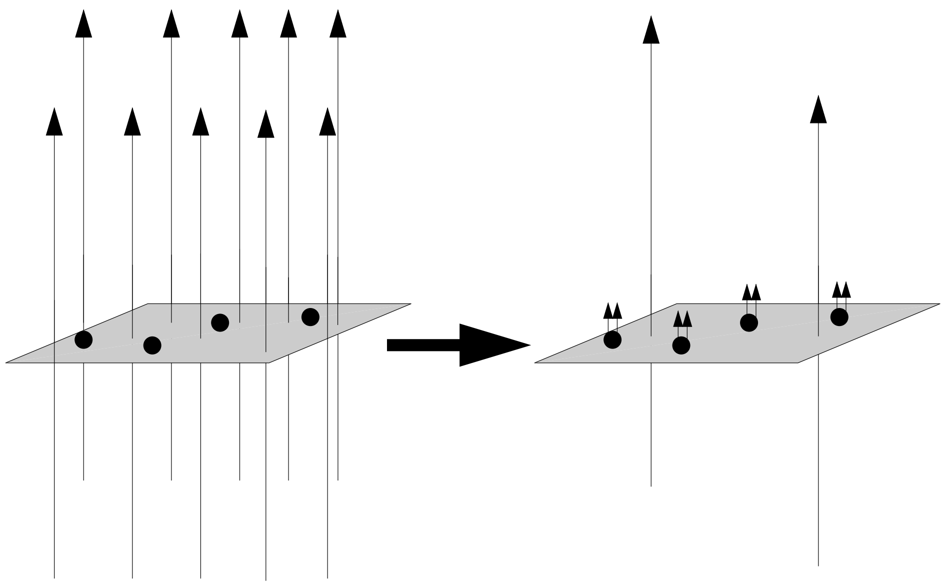

Figure 2. Capturing two flux quanta transforms each electron into a composite fermion that experiences, in effect, a reduced residual magnetic field.

$\nu^* > 1$ is drastically smaller, even when the interaction *between composite fermions* is switched off. For integral values of ν^*, in fact, one gets a *non-degenerate* ground state.

The reduced degeneracy suggests that it should be a good starting point to treat the composite fermions as independent. In this approximation the composite fermions fill a composite-fermion Fermi sea when B^* vanishes ($\nu = 1/2p$), and form composite-fermion Landau levels when it does not. All this action, of course, takes place inside the lowest *electronic* Landau level (Fig 1).

Having identified interacting electrons at filling factor ν with independent composite fermions at ν^*, we write the (unnormalized) microscopic wave functions for interacting electrons at a given ν as:

$$\Psi_\nu = \prod_{j<k} (z_j - z_k)^{2p} \, \phi_{\nu^*} \tag{4}$$

where $z_j = x_j - iy_j$ denotes the position of the jth electron as a complex number, and ϕ_{ν^*} are the the known Slater-determinant wave functions of *non-interacting electrons* at the corresponding ν^*. For simplicity, we have assumed that the electron population is fully polarized, and suppressed the spin part of the wave function.

The wave functions Ψ_ν, which turn out to be an extremely accurate approximation of the actual electron eigenstates, give a precise meaning to the intuitive physics of the "Bohr" description. The factor $\prod_{j<k}(z_j - z_k)^{2p}$ tells us that each electron sees $2p$ vortices at every other electron. That is to say, as the jth electron executes a closed path around the kth, it generates a phase of magnitude $2p \times 2\pi$. By definition, a closed loop around a unit vortex generates a phase of 2π. Thus the factor $\prod_{j<k}(z_j - z_k)^{2p}$ attaches $2p$ vortices to each electron in ϕ_{ν^*}.

So we see that the flux quanta of our "Bohr theory" in fact represent the microscopic vortices of the many-particle wave function, and the composite fermion is actually the bound state of an electron and $2p$ quantum *vortices.* A flux quantum is topologically similar to a vortex, in that it also produces an Aharonov Bohm phase of 2π for a closed path around it. Therefore it is often useful to model the vortices as flux quanta and envision the composite fermion as an electron carrying $2p$ flux quanta.

How do the *vortices* cancel part of the external B? Consider a path in which one particle executes a counter-clockwise loop enclosing an area A, with the other particles held fixed. Equating the sum of the Aharonov-Bohm phase $2\pi BA/\Phi_0$ and the phase $-2\pi 2p\rho A$ coming from the $2p\rho A$ encircled vortices to an effective Aharonov-Bohm phase $2\pi B^* A/\Phi_0$, we get the new field B^* of Eq. (2). Of course, a magnetometer will still measure simply B. But, as far as a composite fermion is concerned, B^* is the real field, as demonstrated by experiments to be discussed below.

The form of the wavefunction Ψ gives an insight into why the repulsive interaction between electrons might force vortices onto them. The wavefunction is very effective in keeping the electrons apart. The probability that two electrons in Ψ will come within a distance r of each other vanishes as $r^{2(2p+1)}$. Contrast that with the r^2 vanishing for a typical state satisfying the Pauli principle. In essence, electrons transmute into composite fermions by capturing $2p$ vortices because this is how they best screen the repulsive Coulomb interaction. The interaction between composite fermions is weak because most of the Coulomb interaction has been screened out, or used up, in making them.

Equations (2), (3), and (4) are the master equations describing the quantum fluid of composite fermions. Since the first two are the same and can be derived from the third, everything ultimately stems from a single equation. The quantum numbers of composite fermion follow straightforwardly from the observation that each one is produced by a single electron. It has the same charge and spin as the electron, and it is also a fermion.

Seeing composite fermions

The crucial, non-perturbative respect in which composite fermions distinguish themselves from electrons is that they experience an effective magnetic field, B^*, that is drastically different from the external magnetic field. The effective magnetic field is so central, direct, and dramatic a consequence of the formation of composite fermions that its observation is tantamount to an observation of the composite fermion itself.

At $\nu < 1$, the experiments clearly reveal the dynamics of composite fermions in B^* rather than electrons subject to B. The most compelling experimental evidence for the composite fermion comes simply from plotting the high-B magnetoresistance as a function of ν^{*-1}, which is proportional to B^*, and noting its striking similarity to the magnetoresistance of electrons at low B (where they are weakly interacting) plotted as a function of ν^{-1}, as shown in Fig. (3). This is a direct evidence that the strongly correlated liquid of interacting electrons at filling factor ν behaves like a weakly interacting gas of composite fermions at ν^*.

The quantum Hall effect is one of the most fascinating phenomena displayed by two dimensional electrons in a magnetic field. One sees plateaus of the Hall resistance R_H with quantized values $R_H = h/fe^2$ centered around $\nu = f$, where f is either an integer ($f = n$) or a simple rational fraction (Fig. 3). The *integral* quantum Hall effect[5] is understood straightforwardly in terms of independent electrons as a consequence of the quantization of the single electron energy into Landau levels, which produces a non-degenerate many-particle ground state whenever ν is an integer n. The analogous integral quantum Hall effect for composite fermions corresponds to $\nu^* = n$. These states occur at fractional *electron* filling factors given by

$$\nu = \frac{n}{2pn \pm 1}\,, \tag{5}$$

which (along with their hole partners $1-\nu$) turn out to be precisely the observed 'magic' fractions at which the fractional quantum Hall effect is observed to be particularly prominent. There is at present evidence for more than 30 fractional quantum Hall states. The equation dictates only odd-denominator fractions, which, with only one exception, is what the experimenters find.

The fractional quantum Hall effect for electrons is thus an integral quantum Hall effect of composite fermions, in effect an observation of composite-fermion Landau levels. This simple explanation of the fractional effect not only obtains all observed fractions in a single step, but also unifies the fractional and the integral quantum Hall effects.

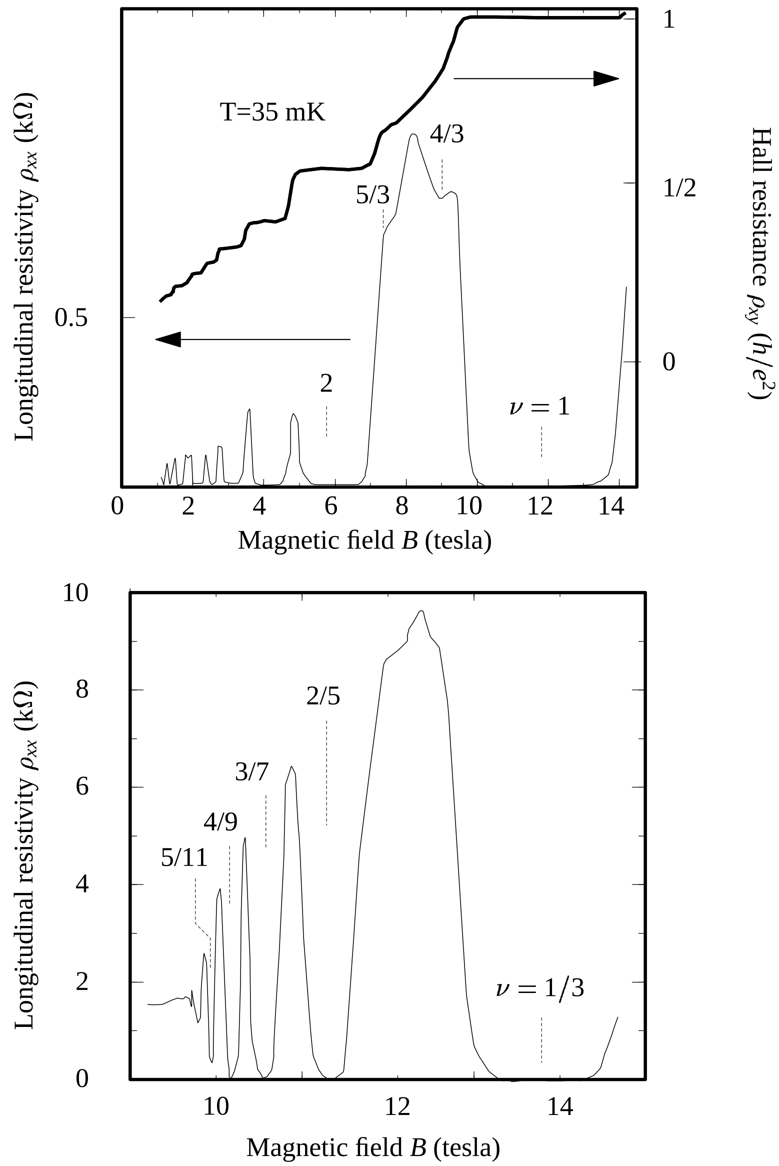

Figure 3. Comparing integral and fractional quantum Hall regimes, top and bottom panels, respectively[3,4]. The thick curve in the top panel is the Hall resistance with quantum plateaus. The thin curves are ordinary longitudinal resistivity with a dip, labeled by its electron filling factor ν, for each plateau. The filling factor $\nu = n/(2n+1)$ corresponds to a composite-fermion filling factor $\nu^\star = n$. Thus the two thin curves, despite their very different electron filling factors are remarkably similar.

The observation of fractional quantum Hall effect serves as a macroscopic confirmation of some of the fundamental postulates of quantum mechanics. The principle governing the surprising precision of the quantization of macroscopic Hall resistance is the single-valuedness of the microscopic many electron wave function, which requires that the vorticity of a composite fermion (the exponent $2p$ in Eq. 4) be precisely quantized to be an integer. The empirical odd denominator rule follows, because the integer must be even so that the many-particle wave function have the exchange anti-symmetry required for fermions.

Robert Laughlin's original theory of $\nu = 1/(2p+1)$ states[5], a subset of the observed fractions, falls naturally within the composite fermion theory. At $\nu^* = 1$, putting into Eq. (4) the explicit form of the non-interacting Slater-determinant wavefunction $\phi_{\nu^*=1}$ yields for the ground state at

$\nu = 1/(2p+1)$

$$\Psi_{1/(2p+1)} = \prod_{j<k}(z_j - z_k)^{2p+1} \exp[-\frac{1}{4}\sum_k \frac{|z_k|^2}{l_B^2}] \tag{6}$$

which is precisely the wave function formulated by Laughlin in 1983 to explain the first observed fractional quantum Hall state ($\nu = 1/3$). (Here, $l_B = \sqrt{\hbar c/eB}$ is the magnetic length.) It represents one filled composite-fermion Landau level.

An early and influential approach, pioneered by Steven Girvin and Allan MacDonald[6] regards the Laughlin wave function as a bose condensate, with the role of boson played by the bound state of an electron and $2p+1$ flux quanta.

What about the fractional quantum Hall effect's celebrated fractional charge? It appears as what is called the "local charge" of an excited composite fermion, defined as the sum of its intrinsic charge ($-e$) and the charge of the screening cloud (or the correlation hole) around it. Its value at $\nu = n/(2pn \pm 1)$ can be obtained from a simple counting argument to be $-e/(2pn \pm 1)$. This fractional charge is a manifestation of a quantized screening by the quantum fluid of composite fermions.

Do composite fermions have a life outside the fractional quantum Hall effect? An important application of the concept concerns the metallic state at $\nu = 1/2$ where no fractional Hall effect is seen. If composite fermions exist at that filling factor, they experience no effective magnetic field ($B^* = 0$). Thus a mean field picture suggests a Fermi sea of composite fermions.

In an influential theoretical work, Bertrand Halperin, Patrick Lee, and Nicholas Read argued that many features of the Fermi surface of composite fermions survive when fluctuations beyond the mean-field theory are taken into account[7]. At ν values near 1/2, the composite fermions, experiencing a very weak magnetic field, would execute classical cyclotron orbits of radius $R^* = \hbar k_F/eB^*$, with the Fermi wave vector given by $k_F = \sqrt{4\pi\rho}$ for a fully polarized Fermi sea in two dimensions. R^* is orders of magnitude larger than any electronic length scale appropriate to B. Experiments by three different groups at Bell Labs and Stony Brook in 1993-94, and several experiments since then, have confirmed that R^* is indeed the cyclotron radius of the charge carriers[3]. Two of these experiment results are shown in Figs. (4) and (5). Farther away from $\nu = 1/2$, the semiclassical orbits of the quantum composite-fermion particles are quantized to produce composite-fermion Landau levels, first exhibiting Shubnikov-de Haas oscillations and then the quantum Hall effect.

The observation of composite fermions in the gapless region around $\nu = 1/2$, where there is neither a quantum Hall effect nor any other sort of excitation gap, was a watershed for the composite fermion concept. It was an explicit demonstration that the composite fermion is more general than its manifestation in the fractional quantum Hall effect, which occurs when pre-existing composite fermions form their Landau levels. The observation of the composite fermion's Fermi sea explicitly verified its Fermi statistics, and the measurement of its cyclotron radius confirmed that it carries a charge $-e$.

How about its spin? The spin degree of freedom is frozen in strong magnetic fields at low temperature, where the Zeeman energy is large compared to the interaction strength and thermal agitation. But in relatively weak fields, several spin polarizations become possible. These differently spin-polarized states, as well as transitions between them as a function of the Zeeman energy (which can be varied without affecting the rest of the dynamics by application of an additional B field parallel to the layer), have been observed, and they are well described in terms of Landau levels of free, spin $1/2$ composite fermions[3,10]. The composite fermion g factor determined from these experiments is close to that of the electron.

For any particle, an important question is: What is its mass? This question is a somewhat subtle one for the composite fermion, because its entire mass is generated dynamically from interactions. There is, after all, no mass parameter in the Hamiltonian of Eq. 1. The mass is most straight-

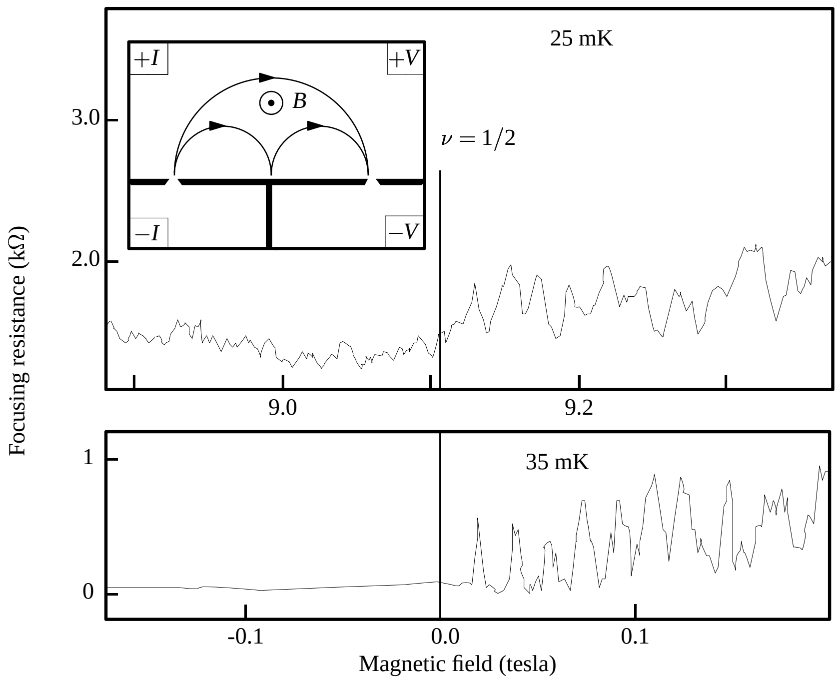

Figure 4. Measuring the effective magnetic field $B^\star$ felt by composite fermions, by magnetically focusing them — injecting them into one constriction and collecting them into another. The lower panel shows the focusing peaks for *electrons* at discrete values of B (near $B = 0$) corresponding to different numbers of bounces (see insert). The upper panel shows the corresponding peaks for composite fermions near $B^\star = 0$. The two sets of peaks (superimposed over mesoscopic resistance fluctuations due to disorder) align when one scales $B^\star$ by a factor of $\sqrt{2}$ to account for the fact that the composite-fermion state, unlike the electron Fermi sea, is spin polarized. (Adapted from Ref. 8).

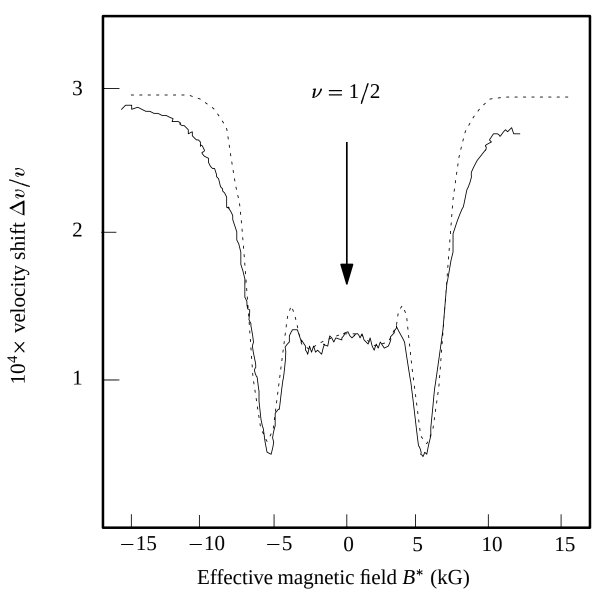

Figure 5. Surface acoustic wave study of the composite-fermion state in the vicinity of $\nu = 1/2$. The velocity shift of surface acoustic waves exhibits principal as well as the secondary resonances, at $|B^*| \approx \pm\, 5.0$ kG and $\pm\, 2.5$ kG, respectively, as expected from theory (dashed curve)[7]. Adapted from Ref. 9.

forwardly determined by measuring the excitation gap at a given filling of composite fermions, and equating it to the cyclotron energy of composite fermions in the B^* field[3]. But it can also be deduced from an analysis of the temperature dependence of the Shubnikov-de Haas oscillations[3], of spin transition experiments[3], or by ascertaining at what Zeeman energy the composite-fermion Fermi sea becomes fully polarized[10]. For typical experimental parameters, the composite-fermion masses obtained from these various methods are on the order of the free electron mass, and much larger than the band mass in say, GaAs, but unrelated to either.

The rich phenomenology of the lowest Landau level liquid thus follows succinctly and coherently from the single principle of the composite fermion, without even the need for a microscopic theory. But the simplicity of the explanation should not obscure the non-trivial nature of the underlying physics: Each strongly interacting electron, with no kinetic energy degree of freedom, captures $2p$ quantum mechanical vortices and is thus magically transformed into a nearly free, massive composite fermion. This composite fermion experiences a magnetic field drastically different from the external one, and its kinetic energy manifests itself through a Fermi sea and Landau levels.

Computer experiments

Fortunately, one can diagonalize the Hamiltonian of Eq. (1) numerically for a *finite* system, to obtain the *exact* eigenfunctions and eigenenergies. That provides further opportunity for rigorous, unbiased, and detailed testing of the composite fermion theory. Some typical energy spectra are shown in Fig. (6). All structure in these spectra is a consequence of the Coulomb interaction, in the absence of which all lowest Landau level states would be strictly degenerate.

The central prediction of the composite fermion theory is that the low-energy physics of interacting electrons in an external magnetic field B resembles that of nearly independent composite fermions in the residual field B^*. This prediction has been extensively confirmed in computer experiments, which establish a one-to-one correspondence between the quantum numbers of the low-energy states of the two systems.

In particular, one expects a gap at filling factor values $\nu = n/(2pn \pm 1)$, which correspond to $\nu^* = n$. Sure enough, at those values the Coulomb interaction removes here the enormous degeneracy of the non-interacting electron system to produce a non-degenerate ground state, as illustrated in Fig. (6). The ground state, circled in each panel, represents n filled Landau levels of composite fermions. The excited states, where dashes and dots coincide, are to be interpreted as different configurations of the composite-fermion exciton (Fig. 1F).

Besides giving the quantum numbers of the low-energy states, the composite fermion theory also yields their wave functions, Ψ, which are now to be projected into the lowest electronic Landau level, as appropriate for the large-B limit under consideration. Extensive studies have shown that these have a nearly perfect overlap with the corresponding exact eigenfunctions, and that they typically predict the energies to within 0.1% or better. Some representative results shown in Fig. (6) and Table I.

To fully appreciate the significance of these comparisons, one should note that, for filled composite-fermion Landau levels or their excitons, Ψ involve no adjustable parameter whatsoever. Furthermore, the actual eigenstates are linear superpositions of a large number of distinct basis states, as indicated by the towers of excitation levels in Fig. 6. That rules out the possibility of accidental agreement. It is rare that such a simple, zero parameter theory for a strongly correlated many-body state has such accuracy and predictive power. These comparisons also demonstrate that the wave functions Ψ go beyond the simple mean field picture that motivated them. They encode the physics of the residual interaction between composite fermions.

Ana Lopez and Eduardo Fradkin have initiated another approach for dealing with corrections to the mean field description, in terms of a Chern-Simons field theory. That approach has been developed further by several groups.[11,7]

A quantitative comparison with the laboratory experiments requires a consideration of Landau level mixing, transverse thickness of the electron wave function, and disorder — all of which were conveniently set to zero in the computer experiments. These realities do not affect the qualitative physics nor the quantizations, but they do introduce various parameters which can only be handled approximately. After incorporating some of these effects in various approximations, we find that theory and experiment typically agree, at present, within $\sim 10\% - 30\%$ for the energy of the neutral composite-fermion exciton[12] and spin related physics[3,10], and within a factor of two for the charged-excitation gap.

Are there other phases?

Do the composite fermions exhibit any other phases? If they were strictly non-interacting, our discussion of their quantum-Hall and Fermi-sea effects would be the end of the story. But there is a residual interaction between composite fermions. By definition, it is whatever is left after most of the Coulomb interaction is used up in giving the composite fermion its mass. The residual

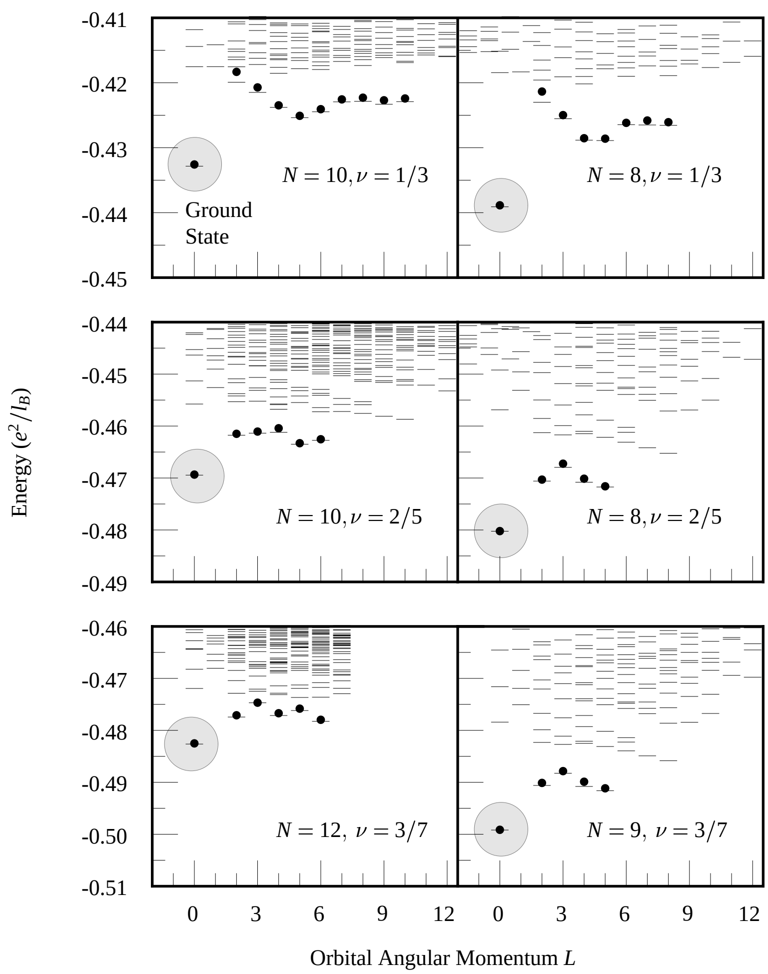

Figure 6. A comparison of exact eigenenergies (dashes) obtained from numerical diagonalization, against the energies predicted by composite fermion theory with no adjustable parameters (dots), for the low-energy states of a model system for $N = 8$ to 12 interacting electrons on the surface of a sphere pierced by a radial magnetic field characterized by filling factor ν. L is the system's total orbital angular momentum (see also the table). The ground state ($L = 0$) is circled. The well-defined branch of low-lying excited states, decorated with dots, represents the composite-fermion exciton in various possible configurations. The l_B in the energy unit is the magnetic length $l_B = \sqrt{\hbar c/eB}$. The towers of excited states extend well beyond the figure's high-energy cutoffs. Adapted from Ref. 13.

ν	N	ground state		excited state	
		composite fermion	exact	composite fermion	exact
1/3	8	-0.4389	-0.4391	-0.4261	-0.4266
	10	-0.4326	-0.4328	-0.4224	-0.4229
2/5	8	-0.4802	-0.4802	-0.4714	-0.4717
	10	-0.4693	-0.4694	-0.4625	-0.4627
3/7	9	-0.4991	-0.4992	-0.4915	-0.4916
	12	-0.4825	-0.4826	-0.4782	-0.4783

Table 1. The exact and the composite-fermion energies per particle for the ground states of the systems in Fig (6). Also given are results for that excited state which has the composite-fermion particle and the composite-fermion hole maximally apart. Adapted from 13.

interaction is often weak enough to be neglected. That is what we have done above. But it might, in certain circumstances, be responsible for creating fascinating new phases. After all, the interaction between electrons — the fermions we know best — generates numerous phases, for example, the BCS superconductor, the Wigner crystal, and Bloch's spontaneously polarized Fermi liquid. There are indications that all of these phases are feasible also for composite fermions.

At sufficiently small ν ($\leq 1/9$), the composite-fermion liquid becomes unstable against a spontaneous generation of excitons, making way for the Wigner crystal[13,14]. In a range of filling factors prior to this ($\nu \leq 1/4$), the composite fermion liquid is predicted to exhibit the Bloch instability, namely a magnetically ordered broken-symmetry phase even in the absence of Zeeman coupling[15].

Another fascinating state is the BCS-like p-wave paired state of fully polarized composite fermions, increasingly believed to be the source of the fractional effect at $\nu = 5/2$, the sole exception to the odd-denominator rule.[16] Here, even though the underlying interaction is purely repulsive, a capture of the vortices during the creation of composite fermions presumably overscreens the Coulomb interaction, producing a weak, effectively *attractive* interaction between them.

Finally, mixed states containing two different flavors of composite fermions (with different numbers of attached flux quanta) would also produce quantum-Hall fractions other than the principal fractions of Eq. (5). Preliminary evidence now exists for such additional fractions.

It remains to clarify the physics of these new phases, of the precise nature of the Fermi liquid at $\nu = 1/2p$, and of the role of disorder in these systems. Another poorly understood issue is how, as the temperature is raised, the composite fermions gradually ionize by shedding their vortices and turn back into electrons.

Quantum particle

Even though the composite fermion behaves, to a great extent, like an ordinary fermion, we must not forget that it is a most unusual particle. First of all, it is a truly collective, many-body entity. The definition of a single composite fermion inherently involves all the particles in the system. Moreover, the composite fermion is a quantum particle. Of course, quantum mechanics describes all particles, but it participates in the very definition of a composite fermion, whose creation is the union of an electron and quantum mechanical phases (vortices). The composite fermion could not exist in a purely classical world. Furthermore, the orbits of composite fermions are quantized to produce a quantum fluid of quantum particles.

Among the remarkable features associated with the physics of composite fermions are the dynamical generation of a mass where there was none to begin with, the quantum-mechanical renormalization of the magnetic field, pairing from purely repulsive interactions, and fractional charge generated by the quantization of screening. It is irresistible to wonder which of these concepts finds wider applications in nature.

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Bibliography

[1] *Composite Fermions*, edited by Olle Heinonen (World Scientific, New York, 1998).

[2] *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).

[3] For a summary of experimental work, see H.L. Stormer and D.C. Tsui in Ref. 2.

[4] H. R. G. Clark *et al.*, *Surface Science* **170**, 141 (1986)

[5] *The Quantum Hall Effect*, Eds. R. E. Prange and S. M. Girvin (Springer Verlag, New York, 1987).

[6] S. M. Girvin in Ref. 5; S. C. Zhang, Int. J. Mod. Phys. B **6**, 25 (1992).

[7] B. I. Halperin in Ref. 2.

[8] V. J. Goldman, *Physica B* **280**, 372 (2000).

[9] R. L. Willett, K. W. West, and L. N. Pfeiffer, *Phys. Rev. Lett.* **75**, 2989 (1995).

[10] S. Melinte, N. Freytag, M. Horvatic, C. Berthier, L. P. Levy, V. Bayot, and M. Shayegan, *Phys. Rev. Lett.* **84**, 354 (2000); I.V. Kukushkin, K. v. Klitzing, and K. Eberl, *Phys. Rev. Lett.* **82**, 3665 (1999).

[11] A. Lopez and E. Fradkin, in Ref. 1. G. Murthy and R. Shankar, in Ref. 1. S.H. Simon in Ref. 1.

[12] V. W. Scarola, K. Park, and J. K. Jain, *Phys. Rev. B,* in press (2000).

[13] J. K. Jain and R. K. Kamilla in Ref. 1.

[14] P. K. Lam and S. M. Girvin, *Phys. Rev. B* **30**, 473 (1984). Hangmo Yi and H. A. Fertig, *Phys. Rev. B* **58**, 4019 (1998).

[15] K. Park and J. K. Jain, *Phys. Rev. Lett.* **83**, 5543 (1999).

[16] W. Pan, J.-S. Xia, V. Shvartx, E. D. Adams, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, *Phys. Rev. Lett.* **83**, 3530 (1999), and references therein.